MA 350 Dr. G. Stoudt
Tenth Reading Assignment

Readings

- Chapter IV (Arabic Primacy with Chinese, Indian, and Maya Contributions) Introduction, page 177

- Biography of al-Khwarizmi, page 199
- Reading 47: From *The Book of Algebra and Almucabola* (Quadratic Equations in Algebra: Verbal Form)-al-Kwarizmi
- Biography of Umar al-Khayyami, page 204
- Reading 48: From the *Algebra*-Umar al-Khayyami
- (just up to end of page 208) Look carefully at the end notes!
- Biography of Marquis Zhang Cang, page 212
- Reading 49: From a Description of the *Nine Chapters on the Mathematical Art* (*Jîuzhang suànshù*)-Li Yan and Dù Shírán

Notes for the Readings

In the Introduction, "A. H." stands for "in the year of the Hegira," after 622 (see page 177).

From Apollonius, *Conics*, Book I:
Proposition 32: If a straight line is drawn through the vertex of a section of a cone, parallel to an ordinate, then it touches the section, and another straight line will not fall into the space between the conic section and this straight line.

Proposition 52 (Problem): Given a straight line in a plane bounded at one point, to find in the plane the section of a cone called parabola, whose diameter is the given straight line, and whose vertex is the end of the straight line, and where whatever straight line is dropped from the section to the diameter at a give angle, will equal in square the rectangle contained by the straight line cut off by it from the vertex of the section and by some other given straight line.

From Euclid, *Elements*:
Proposition V.5: If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole.

Proposition VII.8: If a number be the same parts of a number that a number subtracted is of a number subtracted, the remainder will also be the same parts of the remainder that the whole is of the whole.

Questions for Discussion

Reading 47

1. For the first problem given in Chapter IV (page 201), "a square and 10 roots are equal to 39 units," do the following:
   a. write this as an algebra expression
   b. write a general equation of which this is one type (i.e. use letters *a*, *b*, *c* for coefficients)
   c. using the method described, derive a formula for the solution of the general equation
   d. can you reconcile this with the quadratic formula?
2. On page 202 (right side, first three lines) why do we "take a fourth part of the number 10..."?
3. In the geometrical demonstration, what algebraic technique is being employed? (Use the last paragraph on page 202, with the supplied algebraic notation, to help you.)

Reading 48

1. What is the difference between *Elements* V.5 and VII.8? Why are both needed? Do you see what al-Khayyami is getting at on page 207?
2. What are *Elements* V.5 and VII.8 really saying?
3. On page 208, where al-Khayyami states "It is tangent to the line . . . .," how does he get "tangent" out of *Conics*, Book I, Proposition 32?
4. Be prepared to explain and verify each step in the demonstration on page 208. End note 20 may prove helpful.

Reading 49

1. Compare the area of a circle formula (page 217) with the modern formula and with the Babylonian and Egyptian formulae.
2. What is the method employed on pages 218–219 to solve a system of linear equations called today? Is it perhaps misnamed?
3. There are positive and negative numbers! Check the description on page 220.

Homework Problems

1. Find the intersection of the two parabolae $x^2 = ay$ and $y^2 = bx$ (which is what the problem in reading 48 is all about).

2. (Adapted from Katz, *A History of Mathematics*)
Consider problem 17 from Chapter 7 of *Nine Chapters on the Mathematical Art*:
The price of 1 acre of good land is 300 pieces of gold; the price of 7 acres of bad land is 500. One has purchased altogether 100 acres; the price was 10,000. How much good land was bought and how much bad?

a. Translate this into modern notation. Let x be the amount of good land, y be the amount of bad land.

b. The rule for the solution may have been discovered using the following technique [Katz, p.15], and also "Excess and Deficiency" on page 216:
Guess 20 acres of good land and 80 acres of bad land. This does not solve the problem (it overshoots 10,000). Guess 10 acres of good land and 90 acres of bad land. This does not solve the problem (it undershoots 10,000). Using a notion like slope, and using as the "true" solution, write $\frac{\text{change in } x}{\text{overshoot}} = \frac{\text{change in } x}{\text{undershoot}}$ using modern notation (but keep all expressions positive).

c. Let x_1 and x_2 be the guesses and let b_1 and b_2 be the "over/undershoots." Write the equation from part b. and solve for x to get the formula for the Chinese algorithm

$$x = \frac{b_1 x_2 + b_2 x_1}{b_1 + b_2}.$$

d. What is the solution to the problem using these techniques?

e. Solve the problem using modern techniques.