MA 350 Dr. G. Stoudt
Twelfth Reading Assignment

Readings

- Biography of Cardano, page 261
- From the *Ars Magna* (on the cube of a binomial)-Girolamo Cardano (pp.52-55 Witmer translation)
- Reading 56: From the *Ars Magna*-Girolamo Cardano
- Biography of Viète, page 267
- Reading 57: From *In artem analyticen isagoge* (The New Algebra)-François Viète
- From *The Analytic Art*-François Viète, translated by T. Richard Witmer, Chapter V, On the Rules of Zetetics

- The following five readings are from *The Analytic Art*-François Viète, translated by T. Richard Witmer
- From Ad Logisticem Speciosam Notae Priores (Preliminary Notes on Symbolic Logistic)-François Viète, Proposition I to Proposition XVI
- From Ad Logisticem Speciosam Notae Priores (Preliminary Notes on Symbolic Logistic)-François Viète, Propositions XLVI, XLVIII, XLIX, L
- From Zeteticorum Libri Quinque (Five Books of Zetetica), First Book, Zetetic I and Zetetic II, Fourth Book, Zetetic I
- From De Aequationum Recognitione et Emendatione Tractatus Duo (Two Treatises on the Understanding and Amendment of Equations), De Recognitione Aequationum Tractatus Primerus (First Treatise: On Understanding Equations), Chapter VI, alternate Theorem III
- From De Aequationum Recognitione et Emendatione Tractatus Duo (Two Treatises on the Understanding and Amendment of Equations), De Emendatione Aequationum Tractatus Secundus (Second Treatise:On the Amendment of Equations), Chapter I

- Biography of Stevin, page 273
- Reading 58: From De Thiende (Decimal Fractions)-Simon Stevin

Notes for the Readings

Cardano's diagrams are supposed to represent three dimensional cubes.

From Victor Katz, *A History of Mathematics: An Introduction* (HarperCollins), p. 339:
"For Viète, **zetetic analysis** is the procedure by which one transforms a problem into an equation linking the unknown to various knowns; **poristic analysis** is the procedure exploring the truth of a proposed theorem by appropriate symbolic manipulation; and, finally, **exegetics** is the art of transforming the equation found by zetetics to find a value for the unknown."

Occasionally in Viète you will see something like $A \sim B$. Read this as subtraction, $A - B$.

Questions for Discussion

Cardano Readings

1. Be prepared to verify the steps in the Demonstration on page 263. We will go through this carefully.
2. Write out Cardano's rule (page 264) using variables for the constants: $x^3 + ax = b$.

Chapter V, On the Rules of Zetetics

1. Compare the propositions with those in a modern elementary algebra book. Where do modern texts usually discuss these rules?

Preliminary Notes on Symbolic Logistic-Proposition I to Proposition XVI

1. Be prepared to give modern interpretations of these propositions.

Preliminary Notes on Symbolic Logistic-Propositions XLVI, XLVIII, XLIX, L

1. Write out the algebra performed in Proposition XLVI.
2. In Proposition XLVIII, Viète states "the third triangle is called a triangle of the double angle." What two trigonometric identities are lurking in this construction?
3. What trigonometric identities are lurking in Propositions XLIX and L?

Five Books of Zetetica

1. Rewrite Zetetic I and Zetetic II (First Book) by letting *r* and *s* be the two (unknown) roots.
2. Compare Diophantus II.8 (Reading 46) to Zetetic I (Fourth Book). Solve the example given by Diophantus using Viète's method.

On Understanding Equations

1. Look at Cardano's solution of the cubic (RULE). If we have $A^3 - 3B^2A = B^2D$ with $B > \frac{1}{2}D$, what happens in Cardano's rule? If it helps you, think in modern terms: $x^3 - 3bx = c$, with $c^2 < 4b^3$.
2. Can you see how Viète's trigonometric identities (from Preliminary Notes on Symbolic Logistic) can be used here?

On the Amendment of Equations

1. Here Viète is reducing general cubics (cubes with quadratic and linear affections) to "depressed cubics" (cubes with only a linear affection). Why is this important? (Remember the Cardano solution.)
2. What do you get when you reduce a square with a linear affection to a pure quadratic? What is this technique called today?

Homework Problems

1. Let $x = \sqrt[3]{u} - \sqrt[3]{v}$, $u - v = b$, and $uv = (\frac{a}{3})^3$. Verify that $x^3 + ax = b$.

2. From the Cardano reading you can see that he can solve the "depressed cubic" (no x^2 term) in any of its variations. Viète shows how to turn the general cubic into a depressed cubic, so he could solve all other cubics. Here is the modern version:
In the general cubic $ax^3 + bx^2 + cx + d = 0$, make the substitution $x = y - \frac{b}{3a}$ and simplify, leaving you with a "depressed cubic." Make this substitution and show that it does indeed work.

3. Can you reduce a general quadratic to a "depressed quadratic?" Start with the general quadratic equation $ax^2 + bx + c = 0$, and make the substitution $x = y - \frac{b}{2a}$. Simplify and solve for y. Use the substitution to solve for x. Does what you get look familiar?

4. Solve the following cubic using Cardano's formula (rule): $x^3 + 63x = 316$. Start solving this cubic using Viète's trigonometric method. What problem do you soon run into?

5. Solve the cubic $x^3 - 6x = 4$ by Viète's trigonometric method. Start solving this cubic using Cardano's formula (rule). What is the problem with Cardano's rule in this case?

6. In *The Analytic Art*, Viète shows how "To reduce a cube with a negative linear affection to a square on a solid root minus a square." Viète also notes that "The cube of one-third the coefficient of the affection must be less than one-fourth the square of the constant." He is discussing the equation $A^3 - 3B^p = 2Z^s$. Follow his words and my hints below to solve his problem. (page 289 of Witmer's translation)

Suppose $AE - E^2 = B^p$.
a. Solve this for A.
b. Substitute into the original equation $A^3 - 3B^p = 2Z^s$.
c. Reduce to a sixth degree equation in E which is quadratic in form.
d. Solve for E.

e. Why does Viète say "the cube of one-third the coefficient of the affection must be less than one-fourth the square of the constant?"
f. Solve for A.

7. Repeat #6 a-f for the modern version of the equation, that is $x^3 - 3bx = 2d$. Use modern notation and variables.

8. Solve $x^3 - 81x = 756$ by the method of problem 7.