MA 350 Dr. G. Stoudt
Thirteenth Reading Assignment

Readings

- Chapter VI (The Scientific Revolution at Its Zenith 1620-1720) Introduction, page 291
- Biography of Descartes, page 326
- Reading 62: From *La Géométrie* (Theory of Equations)-René Descartes
- Reading 63: From *La Géométrie* (The Principle of Nonhomogeneity)-René Descartes
- Pages 204-207 of *The Geometry of Rene Descartes*, translated by David Eugene Smith and Marcia L. Latham (Beginning with "Let us apply it...." and ending with "...are the required mean proportionals." Include the footnote!)
- Biography of Fermat, page 341
- "On Analytic Geometry" (extract from *Introduction aux Lieux Plans et Solides*)-Pierre de Fermat, pages 389-396 in *A Source Book in Mathematics*, by David Eugene Smith

Notes for the Readings

Fermat uses notation similar to Viète. Read x for a and y for e.

Questions for Discussion

Reading 62

1. Find a precalculus or college algebra textbook and write down the following to bring to class for discussion:
   a. The Remainder Theorem
   b. The Factor Theorem
   c. The Fundamental Theorem of Algebra
   d. (Descartes) Rule of Signs

2. Can you find all of the items in number 1 in the reading?

Reading 63

1. Viète was concerned with homogeneity of terms, that is you must have something like $A^3 = B^p A$. Descartes is not concerned with homogeneity. How does he "explain" this?

2. Verify that $BE = BD \times BC$ and $BC = BE \div BD$ in Figure 63.1.

3. Verify that $GI = \sqrt{GH}$ in Figure 63.2 (use the Pythagorean Theorem a lot).

4. Verify that $z = OM = \frac{1}{2}a + \sqrt{\frac{1}{4}aa + bb}$ in Figure 63.3.

5. Solve $z^2 = az + bb$ using the quadratic formula. In Figure 63.3, what is the other root, namely $\frac{1}{2}a - \sqrt{\frac{1}{4}aa + bb}$.

6. Read carefully Descartes' Method of Obtaining the Equation of a Curve. We will go through and verify the steps.

From *The Geometry of Rene Descartes*

1. If z and y are two mean proportionals between a and q, what does that mean algebraically (in terms of ratios)?

2. If z is one of the mean proportionals, why is it "evident" that $a : z = z : \frac{z^2}{a} = \frac{z^2}{a} : \frac{z^3}{a^2}$?

3. Using question 1, why must $z^3 = a^2$?

4. What is the latus rectum of a parabola? Where does it appear in the equation of a parabola. Look it up!

5. Follow carefully through the footnote [231] in the reading. We will go through the verification carefully.

"On Analytic Geometry"

1. Where is the idea of coordinate axes first mentioned?

2. Fermat takes the reader through the conic sections. Do you recognize the modern equations of each?

3. Sketch the graph of $xy = c$. Using all that you know about inversions and shifting, sketch the graph of $(x-2)(3-y) = c-6$. Does it look like the second picture on page 391?

4. On the top of page 392, why is it "easy to show" that

$$\frac{NO^2+NO\times OR}{OR^2} = \frac{NZ^2+NZ\times ZI}{ZI^2}?$$

5. Show that $\frac{b^2-a^2}{e^2} =$ a given ratio represents the equation of an ellipse.

6. Show that $\frac{a^2+b^2}{e^2} =$ a given ratio represents the equation of a hyperbola.

Homework Problems

1. Use Descartes rule of signs to determine the number of possible positive and the number of possible negative real zeroes of the polynomial $x^4 - 5x^3 + 5x^2 + 5x - 6$.

2. In the Cardano/Viète homework you reduced a cubic to a depressed cubic and a quadratic to a depressed quadratic. Elsewhere in *La Géométrie* of Descartes (page 376) he states a general rule: (assuming the coefficient of the highest term is 1) "First, we can always remove the second term of an equation by diminishing its true roots by the known quantity of the second term divided by the number of dimensions of the first term, if these two terms have opposite signs; or, if they have like signs, by increasing the roots by the same quantity." What would the substitution be to reduce the following to their depressed forms?

a. $ax^4 + bx^3 + cx^2 + dx + e$

b. $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

c. $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g \pm$

3. Elsewhere in *La Géométrie* of Descartes (page 379) tells how to "multiply or divide all the roots of an equation by a given quantity, without first determining their values." His example involves transforming $x^3 - \sqrt{3}\,x^2 + \frac{27}{27}x - \frac{8}{27\sqrt{3}} = 0$. Descartes says "multiply the second term by $\sqrt{3}$, the third by 3, and the last by $3\sqrt{3}$." He ends up with $y^3 - 3y^2 + \frac{26}{9}y - \frac{8}{9} = 0$. What **substitution** is Descartes really making here?

4. Verify like we did in class for the others that MG is the solution of $z^2 = az - bb$ in Figure 63.4.

5. In Reading 63 (page 339) Descartes states that $yy = cy - \frac{cx}{b}y + ay - ac$ is a hyperbola. Let $a = 1, b = \sqrt{3}$, and $c = 3$. Use rotation of axes to verify that this equation is indeed a hyperbola. Complete the square on the finished result and then sketch the graph. Your calculus book will have a section on rotation of axes—look in the index or the table of contents and see how to do it there.

6. In the Fermat reading (page 395), he states that "The most difficult type of equation is that containing, along with a^2 and e^2, terms involving *ae*, or other given magnitudes, etc." These problems involve rotation of axes (as you saw in problem 5). Fermat's example is $b^2 - 2a^2 = 2ae + e^2$. Find a theorem in your calculus book that uses coefficients of the equation of a conic section to determine the type of conic you have. State the theorem and verify that Fermat's example does give an ellipse as he says.