MA 350 Dr. G. Stoudt
Sixth Reading Assignment

Readings

- Biography of Archimedes, page 131
- Reading 33: From *Sphere and Cylinder* I: Greeting and Assumptions-Archimedes
- Reading 34: From *Sphere and Cylinder* I: Propositions 33 and 34 (Surface and Volume of a Sphere)-Archimedes
- Reading 35: *Measurement of a Circle*: Propositions 1-3 (Approximation of Pi Using in Essence Upper and Lower Sums)-Archimedes (Read Proposition 1 carefully, just read the rest to get the idea.)
- Reading 36: From *Quadrature of the Parabola*: Introduction and Propositions 17-24-Archimedes

Notes for the Readings

from *On the Sphere and Cylinder* I

Proposition 2: Given two unequal magnitudes, it is possible to find two unequal straight lines such that the greater straight line has to the less a ratio less than the greater magnitude has to the less.

Proposition 3: Given two unequal magnitudes and a circle, it is possible to inscribe a polygon in the circle and to describe another about it so that the side of the circumscribed polygon may have to the side of the inscribed polygon a ratio less than that of the greater magnitude to the less.

Proposition 23: The surface of the sphere is greater than the surface described by the revolution of the polygon inscribed in the great circle about the diameter of the great circle.

Proposition 25: [paraphrasing] The surface of a figure inscribed in a sphere is less than four times the greatest circle in the sphere.

Proposition 28: The surface of the figure circumscribed to the given sphere is greater than that of the sphere itself.

Proposition 30: The surface of a figure circumscribed as before about a sphere is greater than four times the great circle of the sphere.

Proposition 32: If a regular polygon with 4*n* sides be inscribed in a great circle of a sphere, as $ab\cdots a'\cdots b'a$, and a similar polygon $AB\cdots A'\cdots B'A$ be described about the great circle, and if the polygons revolve with the great circle about the diameters aa' and AA' respectively, so that they describe the surfaces of solid figures inscribed in and circumscribed to the sphere respectively, then

(1) the surfaces of the circumscribed and inscribed figures are to one another in the duplicate ratio of their sides, and

(2) the figures themselves [i.e. their volumes] are in the triplicate ratio of their sides. [see figure on page 135, Calinger]

from *Quadrature of the Parabola*

Proposition 1: If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis, as *PV*, and if *Qq* be a chord parallel to the tangent to the parabola at *P* and meeting *PV* in *V* then $QV = Vq$. Conversely, if $QV = Vq$, the chord *Qq* will be parallel to the tangent at *P*. (See the figure on the top left of page 143, Calinger).

Proposition 2: If in a parabola *Qq* be a chord parallel to the tangent at *P*, and if a straight line be drawn through *P* which is either itself the axis or parallel to the axis, and which meets *Qq* in *V* and the tangent at *Q* to the parabola in *T*, then $PV = PT$.

Proposition 16: Suppose *Qq* to be the base of a parabolic segment, *q* not being more distant than *Q* from the vertex of the parabola. Draw through *q* the straight line *qE* parallel to the axis of the parabola to meet the tangent at *Q* in *E*. It is required to prove that (area of segment) = $\frac{1}{3}\triangle EqQ$.

Questions for Discussion

Reading 33

1. We have an expression for Assumption 1. What is it? Is it true in every geometry?
2. Draw a picture representing Assumption 2.
3. Where is the Postulate of Archimedes in Eudoxus (Reading 23)?

Reading 34

1. What does Proposition 33 say in terms of π?
2. Look again at Reading 23. What is Archimedes doing in his proof of Proposition 33?
3. What does *a fortiori* mean?

Reading 35

1. Without going into details, what is the basic idea behind how is Archimedes getting his approximation of the ratio of any circle to its diameter?

Reading 36

1. Can you see the Mean Value Theorem in the explanation accompanying Proposition 18?

2. In the explanation accompanying Proposition 19, fill in the details why $PV = \frac{4}{3}RM$ follows from $PV = 4PW$.

3. Where have we seen Proposition 20 and its corollary before?

4. In Propositions 22 and 23, think series. How would we probably state these two propositions?

We will dissect this reading carefully so be ready to supply details to Archimedes' arguments.

Homework Problems

1. Use calculus to find the surface of revolution of the circle and of the inscribed polygon. Use the figure on page 135, that is, use an octagon. Let the radius of the circle be r, and you can center everything at the origin. Note that the octagon touches the circle at 45° intervals.

Note that the area of a regular polygon is $\frac{1}{2}hP$ where h is the apothem and P is the perimeter.

2. Let A be the area of the circle, let K be the area of the triangle, let P_I be the area of the inscribed polygon, and let P_C be the area of the circumscribed polygon.
Write out the two parts (I and II) of the proof of Proposition I using this notation to see if the proof is clearer.

3. Derive a formula for area of a regular n-gon. Next, find the area of both the inscribed and circumscribed polygon (using a circle of radius 1) for polygons of side $n = 6, 12, 24, 48, 96$.