MA 350 Dr. G. Stoudt
Ninth Reading Assignment

Readings

- Biography of Ptolemy, page 166
- Reading 44: From the *Syntaxis* or *Almagest* i (Trigonometry: Table of Sines)-Claudius Ptolemy
- Reading 7.B1, page 245 of *The History of Mathematics: A Reader*, ed. John Fauvel and Jeremy Gray, Regiomontanus on Triangles

Notes for the Readings

From the *Elements* of Euclid:

II.6 If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.

I.47 Pythagorean Theorem

VI. Def. 2 (Note typo on bottom left of page 168, Def. 3 is cited by mistake)
A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so the greater is to the less.

XIII. 9 If the side of the hexagon and that of the decagon inscribed in the same circle be added together, the whole straight line has been cut in extreme and mean ratio, and its greater segment is the side of the hexagon.

IV. 15 Corollary If in a given circle an equilateral and equiangular hexagon is inscribed, the side of the hexagon is equal to the radius of the circle.

XIII. 10 If an equilateral pentagon be inscribed in a circle, the square on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.

III. 21 In a circle the angles in the same segment are equal to one another

VI. 4 In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

VI. 6 If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Questions for Discussion

Reading 44

1. Write up and be ready to discuss the steps in part 32. of the reading (page 168).
2. Find a statement of Ptolemy's Theorem and bring it to class.
3. How can Ptolemy's theorem be considered generalization of the Pythagorean theorem?
4. How does our degree notation (of degrees, minutes, seconds) come from the Babylonians through Ptolemy?

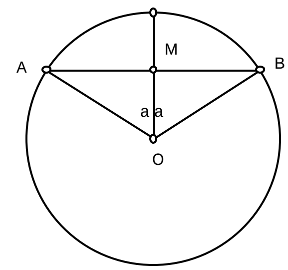
Regiomontanus Reading

1. What is the Law of Sines?
2. How is the Law of Sines proven in a modern trigonometry or precalculus book? What are the similarities/differences? Bring a copy of a modern proof to class.

As an interesting aside, the first occurrence of "the proof is left as an exercise" might be in *De Triangulis Omnimodis* by Regiomontanus, written 1464 and published 1533. He is quoted as saying "This is seen to be the converse of the preceding. Moreover, it has a straightforward proof, as did the preceding. Whereupon I leave it to you for homework."

(Quoted in *Science*, 1994)

Homework Problems



1. Use the diagram to prove that $\sin(a) = \frac{1}{120}\mathrm{Crd}(2a)$ where the radius of the circle is 60 and $\mathrm{Crd}(2a)$ is the length of the chord of the central angle $2a$.

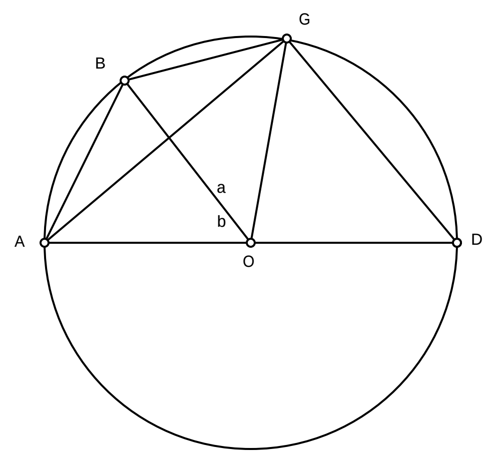
2. Using $\sin(a) = \frac{1}{120}\mathrm{Crd}(2a)$ and any trigonometric identities you know (other than the Pythagorean relation), show that

$$[\mathrm{Crd}(S)]^2 + [\mathrm{Crd}(180 - S)]^2 = 1$$

is equivalent to $\cos^2 x + \sin^2 x = 1$.

3. Using the chords that Ptolemy has found (Crd 36° and Crd 72°) and problem 1, find $\sin 18^\circ$ and $\sin 36^\circ$.

4. Ptolemy states (on page 169) "We shall explain in due course the manner in which the remaining chords obtained by subdivision can be calculated from these, setting out by way of preface this little lemma which is exceedingly useful for the business in hand."
Derive the difference formula
$\sin(a - b) = \sin a \cos b - \cos a \sin b$ using Ptolemy's Theorem. Write in terms of chords first, then use problems 1 and 2 to convert to sines and cosines.



Note that AD is a diameter of the circle,
$\angle AOB = \angle b$, and $\angle AOG = \angle a$.

5. Suppose you have a right triangle $\triangle ABC$ with angles $\angle A$ and $\angle B$ and right angle $\angle C$ and sides opposite these angles a, b, c. If a and $\angle A$ are given, how would you use Ptolemy's chords to solve for b and c? Hint: Find a similar triangle with hypotenuse 120^p inscribed in a semicircle.

6. Find all of the sides and angles of a triangle given that it has sides of length 2 and 3 and the angle opposite the side of length 3 has measure 40°.