Math 643 - Spring 2016  
HW #5 – Additional problems

A) Assume we use the standard 26-letter alphabet (a,b,c, . . . , y, z). Consider the following counting question: “How many 5-letter sequences are possible if each letter is distinct and the letters appear in alphabetical order?” We will answer this question in part (d) below. First, answer the following preliminary questions.

(a) Let \( S \) be the set of all 5-letter sequences for which the 5 letters are distinct. What is \( |S| \)?

(b) Define an equivalence relation, \( \sim \), on set \( S \), where exactly 1 element of each equivalence class would be in alphabetical order. Clearly define \( \sim \), and explain why \( \sim \) is an equivalence relation.

(c) How many elements of \( S \) are in each equivalence class? (They should all be the same size).

(d) Explain how we can use your answers from parts (a) - (c) to answer the original question: “How many 5-letter sequences are possible if each letter is distinct and the letters appear in alphabetical order?”

Note: I realize that this is not the only way to answer this question. You may use other methods to check your answer, but use the approach described here for your primary solution.

B) Consider the set \( T = \mathbb{N} \times \mathbb{N} = \{(a, b) : a, b \in \mathbb{N}\} \). We define a relation on \( T \), which we will label as \( \preceq \), defined as follows: \( (a, b) \preceq (c, d) \) if and only if \( a \leq c \) AND \( b \leq d \). Explain why we can call \( T \) a poset. Use the definition and also provide some examples.

C) Combinatorics Jeopardy. Give an example of a counting question whose answer is the following:

\[ n^n - n! \]