1. Magic Polygons

For those of you who like puzzles like Sudoku, this might be a project for you.

1.1. Definition. A normal (3 × 3) magic square is defined to be an arrangement of the numbers 1 – 9 so that each row, each column, and each of the two main diagonals sum to the same total, called the magic sum. A general 3 × 3 magic square allows for the arrangement of any 9 distinct numbers so that each row, each column, and each of the two main diagonals sum to the same total.

The definition of a magic square can be generalized to normal n × n magic squares, but this would mean arrangements of the numbers 1 – n². However, to define and study magic polygons, we will stick to the idea of a 3 × 3 magic square.

1.2. Example. The following is a normal 3 × 3 magic square:

\[
\begin{array}{ccc}
2 & 7 & 6 \\
9 & 5 & 1 \\
4 & 3 & 8 \\
\end{array}
\]

In this example, each of the rows, each of the columns, and both of the diagonals add to the magic sum of 15.

Another way to think of forming the “board” for a magic square is to start with a square:

Then place a square at each corner and the center of the square, drawing a line segment to connect opposing corners:
Finally, place a square on the midpoint of each edge of the square, drawing a line segment to connect each of these new squares to the square opposite of it:

To generalize this, consider a $2n$-sided polygon for any $n \in \mathbb{N}$. Then the same construction can be made, following the three steps below:

1. Draw the $2n$-sided polygon
2. Place a square at each corner and at the center of the $2n$-sided polygon, drawing line segments to connect opposing corners.
3. Place a square at the midpoint of each edge of the $2n$-sided polygon, drawing line segments to connect each of these new squares to the square opposite of it.

Let’s illustrate this process for a hexagon (a 6-sided polygon):

1. Draw the hexagon
(2) Place a square at each corner and at the center of the hexagon, while connecting opposite corners with line segments.

(3) Place a square at the midpoint of each edge of the hexagon, drawing line segments to connect each of these new squares to the square opposite of it.
Is it possible to fill in each square with a distinct number and have the sum of each edge and each diagonal to be the same number, the \textit{magic sum}? For a normal $n \times n$ magic square, the magic sum can be computed as:

$$m = \frac{n(n^2 + 1)}{2}.$$ 

When thinking about "magic polygons", this leads to the following questions:
(a) For what values of $n$ does a magic polygon exist?
(b) Is there a formula for the magic sum for the cases where a magic polygon exists?
(c) The idea of this write-up only includes even-sided polygons, can you define what a magic polygon might look like for an odd-sided polygon?