Mutual Adjustments between Process and Form in a Desert Mountain Fluvial System

Bruce L. Rhoads

Department of Geography, University of Illinois at Urbana-Champaign, Urbana, IL 61801

Abstract. The analysis of earth surface systems is complicated by mutual adjustments among geomorphic variables. Most statistical models used by geomorphologists implicitly assume unidirectional causation. Simultaneous-equation models represent an alternative statistical approach by which mutual adjustment mechanisms can be analyzed explicitly. This technique is used to develop a process-response model of a small desert mountain fluvial system in order to examine the system’s internal structure and dynamics. In general, results confirm the expected interactions among variables in the model. Discharge, size of bed material, and the type of bank material are the dominant factors directly influencing channel width, suggesting that channel form reflects an interaction among hydrologic conditions, the caliber of the sediment load, and the resistance imposed by the channel perimeter. Results also indicate that local and upstream energy conditions strongly influence the mean grain size and sorting of the streambed materials at a particular location in the drainage network, but that bed material caliber has a relatively weak effect on channel gradient. Although the model contains a self-regulatory feedback loop, the dynamics of this fluvial system appear to differ from the concept of steady-state equilibrium. When displaced from stability by an exogenous disturbance, the variable states do not return to or fluctuate around their initial values. Rather, they converge on new limits that are different from their previous stable values. This type of response suggests that the system is metastable. The process-response model provides a comparative framework for future investigations of desert mountain fluvial systems.

Key Words: fluvial geomorphology, simultaneous-equation models, systems analysis, equilibrium.

Although geomorphologists have readily embraced systems concepts (Strahler 1952; Chorley 1962; Huggett 1985), they have been slow to adopt sophisticated forms of systems analysis (Hart 1986, 122). This situation partly reflects a limited understanding of cause-and-effect relationships in geomorphology. The surface of the earth is a complex process-response system. Flows of energy and matter through this system drive physical processes that act upon and change the morphologic characteristics of the landscape. These changes, in turn, modify the physical processes. Thus, earth surface processes and forms interact with one another and are mutually dependent.

A traditional approach to the analysis of geomorphic systems involves the deductive formulation of bivariate or multivariate linear regression models (Chorley and Kennedy 1971; Huggett 1985). This approach attempts to simplify complex reality by isolating relationships among major components of a real-world system. However, if significant feedbacks occur among the system components, the results of such analyses will be inaccurate and misleading because the regression model is misspecified; it does not accurately represent the true underlying causal structure of the real-world system (Hanushek and Jackson 1977, 79–80). The consequences of model misspecification are biased, inconsistent parameter estimates and the invalidation of inferential statistical tests.

The analysis of complex earth surface systems requires methods that can handle mutual adjustments among geomorphic variables.
Figure 1. Diagram illustrating (a) a hierarchical causal system and (b) a nonhierarchical causal system. Variables represented by X are exogenous, those represented by Y are endogenous.

\[ Y_1 = a_1 + \gamma_{11}X_1 + \epsilon_1 \]  
\[ Y_2 = a_2 + \beta_{21}Y_1 + \gamma_{21}X_1 + \epsilon_2 \]  
\[ Y_3 = a_3 + \beta_{31}Y_1 + \beta_{32}Y_2 + \epsilon_3 \]  

Simultaneous-equation models represent such an approach. These models were originally developed by economists (Haavelmo 1943). Subsequently, they have been adopted by political and social scientists (Blalock 1971) as well as by human geographers (Gober 1978; Cadwallader 1982). Only recently have they emerged in geomorphology (Miller 1984). This paper uses the simultaneous-equation approach to explore the interactions among process and form within a small desert mountain fluvial system. More specifically, the research addresses the questions: what interactions occur among stream channel morphology, bed material, hydrologic regime, and hydraulic processes in desert mountain fluvial systems, and what do these interactions imply concerning the process-response behavior of these systems?

**Methodology: Simultaneous-Equation Models**

Simultaneous-equation models represent a form of linear regression analysis by which reciprocal causation (i.e., feedbacks) among variables can be analyzed explicitly. A brief overview of the technique is presented below. Detailed discussions of these models are provided by Hanushek and Jackson (1977), Namboodiri, Carter, and Blalock (1975), Asher (1983), and Johnston (1984).

The empirical analysis of a real-world system begins with the formulation of a model that represents the hypothesized causal structure of the system. Causal models are commonly expressed schematically as path diagrams and mathematically as sets of structural equations (Figure 1). In hierarchical models, the causal linkages among the variables are unidirectional, and higher-ordered endogenous (dependent) variables do not appear as explanatory variables in lower-ordered structural equations. In most cases, these models can be meaningfully estimated by ordinary least squares (OLS) because they do not inherently violate any assumptions of OLS. Nonhierarchical, simultaneous-equation models, on the other hand, are characterized by reciprocal causation in the form of feedback linkages. At least one of the structural equations contains a higher-ordered endogenous variable as an explanatory variable.

The meaningful estimation of simultaneous-equation models depends upon whether or not a unique solution (or finite number of solutions) exists for each coefficient in the structural equations. This issue is referred to as the identification problem; it focuses on the relation-
ship between the structural equations and the reduced-form of these equations. The reduced-form is generated by substituting for each endogenous variable on the right-hand side of each structural equation, the expression that defines it in the structural equations. The resulting equations contain a single endogenous variable expressed as a function of the exogenous (independent) variables only. A structural equation is identified if its structural coefficients can be determined from mathematical transformations of the reduced-form coefficients. If the reduced-form expressions provide a unique solution for each structural coefficient, the equation is exactly identified. If more than one solution exists, the equation is overidentified. A simultaneous-equations model is identified if and only if each of its component equations is exactly identified or overidentified. Model underidentification occurs when there are more unknown structural parameters than reduced-form expressions. In this situation, finite solutions for the structural coefficients cannot be derived from the reduced-form coefficients.

Two procedures are used to test for identification. The order condition states that each structural equation must exclude at least \( k - 1 \) exogenous variables, where \( k \) is the number of endogenous variables included in a given equation (Asher 1983). The simultaneous-equation model in Figure 1 is underidentified because Equation 4 includes two endogenous variables \( (Y_1, Y_2) \), but does not exclude any exogenous variables. This test is easy to apply, but it does not guarantee that an equation will be identified. The rank condition, on the other hand, is both a necessary and sufficient condition for identification. This condition states that in a model of \( k \) linear equations, an equation is identified if and only if the matrix of structural coefficients still has at least one nonzero determinant of \( k - 1 \) rows after eliminating columns from the matrix that contain nonzero entries for the equation in question as well as the row representing this equation (Asher 1983). Calculating the rank of a matrix is normally a tedious procedure, making this test difficult to apply. Recently, Berry (1983) developed a simple algorithm that is mathematically equivalent to the rank condition. His procedure greatly facilitates testing for this condition.

Identified or overidentified simultaneous-equation models pose an estimation problem because they inherently violate the OLS assumption that the error terms must be uncorrelated with each explanatory variable in a given equation (Hanushek and Jackson 1977, 166–67). For example, to apply OLS correctly to Equation 4 in Figure 1, one must assume that \( e_i \) is uncorrelated with \( Y_i \) and \( X_i \). However, because \( e_i \) influences \( Y_i \), and \( Y_i \) in turn affects \( Y_2 \) (Equation 5), \( Y_i \) and \( e_i \) will be correlated, violating the error assumption. Even if this equation were identified, the estimates of the coefficients obtained from OLS would be biased and inconsistent.

A variety of estimation procedures are available for exactly identified or overidentified simultaneous-equation models, but the simplest and most widely-used method is two-stage least squares (2SLS). This technique involves the use of instrumental variables which are generated by regressing each endogenous variable in the model against the entire set of exogenous variables. The predicted values of the endogenous variables are then used as instruments in a second-stage of the regression to estimate the coefficients in the structural equations. Ideally, the instruments from the first stage should be uncorrelated with the error terms of the structural equations, thereby satisfying the OLS error assumption. But the estimated values usually are not totally independent of the residual variance in finite samples (Hanushek and Jackson 1977, 235). Therefore, two-stage least squares yields biased but consistent estimates of the structural coefficients. It also efficiently incorporates the excess information in overidentified models: 2SLS estimates of overidentified coefficients are weighted averages of the separate estimates obtainable from the reduced-form coefficients. Despite the undesirability of bias, 2SLS estimates represent an improvement over biased and inconsistent OLS estimates.

**Study Area**

The desert mountain fluvial system selected for analysis is a small (6.8 km²) drainage basin
located at the southern end of the McDowell Mountains in south-central Arizona (Figure 2). This mountain drainage system is representative of others in the southern part of the Basin and Range in terms of catchment area, topography, stream channel characteristics, geology, vegetation, and climate. The terrain in the basin is rugged with many hillslopes exceeding 30 degrees. Stream channel gradients range from 3 percent at the mouth of the watershed to over 20 percent near the drainage divide. The headwater reaches of these ephemeral streams lie within narrow valleys comprised of bedrock or calcified colluvium. Channels in the mid- and lower portions of the basin incise coarse alluvium to form arroyos. Such entrenchment is characteristic of mountain streams throughout southern Arizona (Melton 1965; Pêwé and Schank 1973; Wells 1977). Streambed material consists primarily of quartzite, which underlies the majority of the basin (Figure 3). Granitic rocks occur locally at two locations, but only the granodiorite contributes significant amounts of sediment to the stream channels. Vegetation types include paloverde, mesquite, creosote bush, ocotillo, saguaro, and several other types of cacti and shrubs (Turner 1974). The climate is arid subtropical with a mean annual temperature of approximately 20 °C and mean annual precipitation of about 300 mm (Christenson, et al. 1978). The annual precipitation regime has two maxima during the year. The primary peak occurs in July and is associated with rainfall from thunderstorms. This type of precipitation can occasionally produce flash floods within small (<260 km²) watersheds in the Southwest. A secondary peak is associated with the passage of cyclonic storms during December and January. These events usually generate only minor runoff from small drainage basins in southern Arizona (Osborn 1983).

The Conceptual Model

The conceptual process-response model for the McDowell Mountain fluvial system consists of eleven variables, five of which are exogenous and six of which are endogenous (Figure 4, Table 1). The exogenous variables, with the exception of UPSTREAM, represent environ-
mental controls that influence the forms and processes of the stream channels. In other words, the channels constitute a dependent subsystem nested within the drainage basin system. Such a structure is consistent with Schumm and Lichty’s (1965) view of river systems during the modern time span. Although Schumm and Lichty (1965, table 2) included vegetation and climate as independent variables on this time scale, these factors are essentially constant throughout the study area. The exogenous variable, UPSTREAM (Table 1), adds a dynamic component to the model in which the stream channels are viewed as gravitationally-driven conveyors of water and sediment; it indicates how the sediment transport conditions at an upstream location influence the bed material properties in the reach immediately downstream. Thus, the model assumes that over the modern time span sedimentary influences operate primarily in the downstream rather than upstream direction.

The hypothesized structure of the channel subsystem contains feedback loops that include measures of fluvial processes and fluvial forms (Figure 4); therefore, the model represents a process-response system (Chorley and Kennedy 1971). This causal structure is based on relationships established by previous geomorphic research. These relationships are discussed below.

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**Table 1. Definitions of Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous Variables</td>
<td></td>
</tr>
<tr>
<td>AREA</td>
<td>Drainage area (km²)</td>
</tr>
<tr>
<td>RELIEF</td>
<td>Relief ratio = H/L where H is the subbasin relief and L is the basin length measured from the sample site to the drainage divide (dimensionless)</td>
</tr>
<tr>
<td>BANKTYPE</td>
<td>Dummy variable –1 for arroyos (0 in logarithmic units), and 2.72 for channels flanked by bedrock valley sides (1 in logarithmic units)</td>
</tr>
<tr>
<td>UPSTREAM</td>
<td>$\tau_u / \tau_{cr}$, where $\tau_u$ is the shear stress at the sample site immediately upstream and $\tau_{cr}$ is the critical shear stress required to move the mean grain size of the bed material at the sample site immediately upstream. $\tau_u$ is computed using Shields’s entrainment function (Richards 1982, 82). The ratio, $\tau_u / \tau_{cr}$, which measures excess shear stress, is related to bed material transport (Graf 1971; Kalinske 1947) (dimensionless)</td>
</tr>
<tr>
<td>ROCKTYPE</td>
<td>Dummy variable –1 for sites located downstream from quartzite source areas (0 in logarithmic units), 2.72 for sites located downstream from the granodiorite sediment source area (1 in logarithmic units)</td>
</tr>
<tr>
<td>Endogenous Variables</td>
<td></td>
</tr>
<tr>
<td>DISCH</td>
<td>Estimated discharge of the 25-year flood (m³/s)</td>
</tr>
<tr>
<td>SLOPE</td>
<td>$\sin \theta$, where $\theta$ equals the angle of inclination of the channelbed (dimensionless)</td>
</tr>
<tr>
<td>WIDTH</td>
<td>Width of the active stream channel (m)</td>
</tr>
<tr>
<td>STRESS</td>
<td>Mean bed shear stress of the 25-year flood = $\gamma R SLOPE$ (N/m²), where $\gamma$ is the specific weight of water and sediment (11,000 N/m³) and R is the hydraulic radius of flow (m)</td>
</tr>
<tr>
<td>MEAN</td>
<td>Mean grain size of bed material (millimeters)</td>
</tr>
<tr>
<td>SORT</td>
<td>Degree of sorting of bed material (standard deviation of the particle size distribution) (phi units)</td>
</tr>
</tbody>
</table>
Table 2. Descriptive Statistics for Variables in the Model (Log-transformed Data)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
<th>min.</th>
<th>max.</th>
<th>skew.</th>
<th>kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AREA</td>
<td>-0.56</td>
<td>1.05</td>
<td>-2.45</td>
<td>1.90</td>
<td>0.32</td>
<td>-0.80</td>
</tr>
<tr>
<td>BANKTYPE</td>
<td>0.29</td>
<td>0.46</td>
<td>0.00</td>
<td>1.00</td>
<td>0.93</td>
<td>-1.15</td>
</tr>
<tr>
<td>DISCH</td>
<td>1.91</td>
<td>0.58</td>
<td>0.64</td>
<td>3.12</td>
<td>-0.06</td>
<td>-0.59</td>
</tr>
<tr>
<td>MEAN</td>
<td>3.07</td>
<td>0.55</td>
<td>1.61</td>
<td>4.14</td>
<td>-0.60</td>
<td>-0.09</td>
</tr>
<tr>
<td>RELIEF</td>
<td>-1.64</td>
<td>0.35</td>
<td>-2.52</td>
<td>-0.89</td>
<td>-0.16</td>
<td>-0.66</td>
</tr>
<tr>
<td>ROCKTYPE</td>
<td>0.26</td>
<td>0.44</td>
<td>0.00</td>
<td>1.00</td>
<td>1.09</td>
<td>-0.83</td>
</tr>
<tr>
<td>SLOPE</td>
<td>-3.18</td>
<td>0.61</td>
<td>-5.12</td>
<td>-1.91</td>
<td>-0.26</td>
<td>0.38</td>
</tr>
<tr>
<td>SORT</td>
<td>0.64</td>
<td>0.23</td>
<td>0.00</td>
<td>1.08</td>
<td>-0.60</td>
<td>-0.09</td>
</tr>
<tr>
<td>STRESS</td>
<td>5.44</td>
<td>0.59</td>
<td>3.76</td>
<td>6.78</td>
<td>-0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>UPSTREAM</td>
<td>2.44</td>
<td>0.74</td>
<td>0.72</td>
<td>4.40</td>
<td>0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>WIDTH</td>
<td>1.32</td>
<td>0.56</td>
<td>0.00</td>
<td>2.94</td>
<td>0.24</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

The 25-year flood (DISCH) serves as the discharge component for the study because it is large enough and occurs frequently enough to perform significant amounts of geomorphic work. Normally, the bankfull discharge, which has a recurrence interval of 1.5 years for perennial streams, is viewed as the dominant event controlling channel form. But in arid basins, upstream transmission losses may reduce the magnitude of a high frequency event (such as the 1.5-year flood) at a given location to the point where it is too small to significantly affect channel morphology (Reich and Renard 1981). More important, large, infrequent floods tend to have relatively greater geomorphic importance in coarse-bedded ephemeral streams (Baker 1977; Wolman and Gerson 1978).

The formative discharge is usually considered an independent variable in the fluvial system (Schumm and Lichty 1965). But, as discussed above, transmission losses in ephemeral streams strongly affect the magnitude of a flood of a certain recurrence interval (Keppel and Renard 1962). The rate of transmission loss within a specific reach primarily depends on channel width and the volume and caliber of the channel alluvium (Renard 1970). Only the former variable is considered in this study because transmission losses had to be estimated with a hydrologic model (discussed below) that uses a constant rate of infiltration for coarse bed material.

The gradient of a stream (SLOPE) is usually regarded as a dependent variable in the fluvial system that is determined by mean discharge, sediment load, sediment caliber, and channel form (e.g., Mackin 1948; Rubey 1952; Hack 1957; Cherkauer 1972). Although it was not possible to obtain direct measurements of sediment load, this variable is represented indirectly by the relief ratio (RELIEF) (Schumm 1954). The width of the active channel bed (WIDTH) served as a surrogate for width/depth ratio, the measure commonly used to represent channel form, because the bankfull depths of arroyos and narrow mountain streams in the study area were often indeterminate.

Recently the dependency of channel gradient on bed material caliber has been questioned, particularly for coarse-bedded upland streams. Ferguson (1981) and Knighton (1984, 158–59) pointed out that in many cases mean grain size (sediment caliber) may be adjusted to channel gradient rather than vice versa. In other words, gradient may indirectly affect grain size through its influence on shear stress (STRESS). These alternative hypotheses (SLOPE-STRESS-MEAN versus MEAN-SLOPE) are included in the causal structure of the process-response model (Figure 4).

According to Knighton (1984, 104), the dominant controls on stream channel morphology (WIDTH) are discharge (DISCH), the caliber of the bed-material load (MEAN), and the composition of the channel boundary (BANKTYPE). Channel gradient (SLOPE) may also influence the cross-sectional form of steep, mountain streams because high gradients promote rapid runoff and reduced channel storage (Miller 1958; Day 1972).

Shear stress (STRESS) is directly related by definition to channel slope (SLOPE) (Table 1). STRESS also depends on channel form (WIDTH) and the amount of streamflow (DISCH). For constant discharge and gradient, increases in channel width will reduce the depth of flow,
resulting in a lower mean shear stress, whereas for constant width and slope, increases in discharge will increase flow depth and shear stress.

The mean grain size (MEAN) and sorting (SORT) of bed material are controlled by the characteristics of the parent materials (BANK-TYPE, ROCKTYPE) and by the energy available to modify these materials through selective sediment transport and abrasion (STRESS) (Folk 1974). The degree of sorting also depends on the inertia or caliber of the bed material (MEAN). In addition to local energy conditions, the upstream sediment transport rate (UPSTREAM) may also influence MEAN and SORT. Increases in this rate may increase or decrease mean grain size and improve or worsen the degree of sorting, depending on whether such increases enhance the downstream movement of fine bedload or coarse bedload.

Data

Data on stream channel characteristics were collected at 149 sites scattered randomly throughout the drainage network of the McDowell Mountain watershed (Figure 5) (Table 2). At each site, one-hundred particles were sampled from the surface of the active streambed, using the grid method devised by Wolman (1954), and the dimensions and gradient of the stream channel were surveyed with an Abney level and optical rangefinder. The active streambed consisted of alluvial materials having a fresh appearance and lacking surficial weathering or varnish layers. Mean grain size (MEAN) and sorting (SORT) of the samples were determined graphically from cumulative frequency curves using formulas presented by Folk (1974). Measurements with a polar planimeter and opisometer on enlarged U.S. Geological Survey topographic maps (scale 1:6000) provided data on the morphometric variables (AREA, RELIEF).

Because no streamflow data were available for the study basin, discharges had to be estimated using a hydrologic model. Such an approach has yielded meaningful results from other geomorphologic studies conducted in remote areas of the Southwest (Graf 1979, 1982, 1983a). The primary considerations in selecting a model for this research were that it be easy to apply, require data that are readily obtainable, and account for the major processes affecting waterflow in arid-region streams. The distributed model developed by Lane (1982) best meets these criteria. It represents a compromise between complex, physically-based models and simplified statistical procedures. Most important, it provides reasonable first-order estimates of discharges of various frequencies for small (<50 km²) watersheds in southern Arizona (Lane 1982, 1985). The model links a modified version of the SCS curve number method (Soil Conservation Service 1972) with a transmission loss function (Lane 1980). According to table 2 in Lane (1982), all channel segments in the study area are in the very high transmission loss category (median grain size >2 mm) with a corresponding effective hydraulic conductivity of >12.7 cm/hr. Lane does not provide a functional relationship between size of bed material and hydraulic conductivity for this transmission loss category; therefore, it was assumed that the effective hydraulic conductivity for each channel reach equaled 13 cm/hr.

The discharges estimated by the distributed model represent hillslope runoff hydrographs modified by the transmission loss function. The flow characteristics at a particular site depend not only on discharge, but also on the cross-sectional form of the stream channel. A computer algorithm calculated shear stress (STRESS) for each sample site based on Golubstov's (1969) resistance formula for steep mountain streams.
and the surveyed dimensions of the stream channel cross-section. The program computed these values by iteratively adjusting the water surface elevation within a fixed cross-section until the computed discharge converged on the value estimated with the hydrologic model. Additional details on this computer program can be found in Rhoads (1987).

The Structural Model

Six equations describe the structural representation of the conceptual process-response model (Figure 6, Table 3). Previous studies indicated that all relationships are nonlinear and can best be described by multivariate power functions. Inspection of bivariate scatterplots confirmed the appropriateness of using log-transformed values. Equations 7–11 (Table 3) display a nonhierarchical structure, and the algorithm devised by Berry (1983) was used to determine whether each of these equations satisfies the rank condition for identification. Results indicated that all of the equations are overidentified; therefore, two-stage least squares was employed to estimate their structural coefficients. In the estimation procedure, the value of the SLOPE-STRESS coefficient ($\beta_{42}$) was restricted to 1.0. Because SORT has no feedback effect on the system (Figure 6), Equation 12 can be isolated for analytical purposes (Namboodiri et al. 1975, 526–30). Coefficients for this hierarchical equation were estimated by ordinary least squares.

Evaluation of the Results

In general, statistical results confirm the hypothesized structure of the conceptual process-response model. All but one ($\gamma_{63}$) of the coefficients have the expected sign and only a few are not significant at the .05 level (Table 4). The standardized values of the coefficients indicate the relative importance of the causal connections, whereas the unstandardized estimates indicate the percentage change in the dependent variable for a percentage change in the explanatory variable (Hanushek and Jackson 1977, 98). For purposes of comparison, the ordinary least squares (OLS) estimates for Equations 7–11 are also presented in Table 4. Although the significance levels of the OLS estimates are fairly consistent with those for 2SLS, the magnitudes of several of the OLS coefficients differ considerably from the 2SLS estimates. For example, one might conclude from the OLS results that channel width (WIDTH) has a relatively minor influence on channel gradient (SLOPE) (Equation 8). Such a difference illustrates how substantive conclusions depend upon the choice of statistical technique. In this study, one must place greater confidence in the 2SLS results because the nonhierarchical structure of the process-response model implies that the OLS estimates are biased and inconsistent. These results are discussed below.

As expected, RELIEF, AREA, and WIDTH all have significant effects on channel gradient (SLOPE), but the caliber of the bed material

![Figure 6. Structural process-response model.](image-url)
(MEAN) does not appear to influence SLOPE strongly. The latter result suggests that mean grain size may be adjusted directly to prevailing energy conditions (SLOPE-STRESS-MEAN) rather than vice versa (MEAN-SLOPE). Although there is a statistical basis for the removal of MEAN from Equation 8, physical considerations suggest that it be retained. The model should be tested in a wide variety of fluvial environments before any variables are excluded from it. In any regression analysis, a physically relevant variable may be insignificant due to multicollinearity or to insufficient variance in the sample data, or, conversely, a physically irrelevant variable may be significant via a sampling fluke. But it should be noted that MEAN is not highly correlated with the other explanatory variables in Equation 8 (Table 5) nor is the variance for mean grain size exceptionally small (Table 2).

Stream width (WIDTH) is significantly influenced by discharge (DISCH), the type of bank material (BANKTYPE), and the mean grain size of the bed material (MEAN). The standardized coefficients for Equation 9 indicate that discharge (DISCH) and sediment caliber (MEAN)
Table 5. Correlation Matrix for Explanatory Variables in the Process-Response Model (Log-
transformed data)

<table>
<thead>
<tr>
<th></th>
<th>AREA</th>
<th>RELIEF</th>
<th>SLOPE</th>
<th>DISCH</th>
<th>WIDTH</th>
<th>STRESS</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>AREA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RELIEF</td>
<td>-.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SLOPE</td>
<td>-.66</td>
<td>.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DISCH</td>
<td>.92</td>
<td>-.37</td>
<td>-.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WIDTH</td>
<td>.82</td>
<td>-.49</td>
<td>-.48</td>
<td>.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STRESS</td>
<td>-.50</td>
<td>.63</td>
<td>.93</td>
<td>-.35</td>
<td>-.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEAN</td>
<td>-.37</td>
<td>.04</td>
<td>.24</td>
<td>-.23</td>
<td>-.34</td>
<td>.24</td>
<td></td>
</tr>
<tr>
<td>UPSTREAM</td>
<td>-.15</td>
<td>.51</td>
<td>.49</td>
<td>-.16</td>
<td>.03</td>
<td>.47</td>
<td>-.36</td>
</tr>
<tr>
<td>BANKTYPE</td>
<td>-.03</td>
<td>.10</td>
<td>.10</td>
<td>.00</td>
<td>-.20</td>
<td>.21</td>
<td>-.10</td>
</tr>
<tr>
<td>ROCKTYPE</td>
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<td>.14</td>
<td>-.05</td>
<td>.11</td>
<td>.23</td>
<td>-.08</td>
<td>-.64</td>
</tr>
</tbody>
</table>

have the strongest influences on channel morphology, suggesting that channel form is mainly the product of an interaction between the dominant discharge and the resistance of the perimeter sediment. Bank material (BANKTYPE) also plays an important role in determining the morphology of these mountain streams, with channels flanked by bedrock valley sides tending to be narrower than those within arroyos. Although the coefficient for channel gradient (SLOPE) in Equation 9 has the expected negative sign, its magnitude is not significant at the .05 level. Comparison of the standardized values for coefficients $\beta_{23}$ and $\beta_{32}$ indicates that channel width influences channel gradient more strongly than channel gradient influences channel width.

Shear stress (STRESS), lithology (ROCKTYPE), the type of bank material (BANKTYPE), and the upstream transport rate (UPSTREAM) all have significant effects on the caliber of the channelbed material (MEAN). Shear stress, which reflects stream competence, is the dominant control, with grain size diminishing as competence declines. The negative value for the UPSTREAM coefficient indicates that increases in the upstream transport rate reduce the downstream mean grain size. This result implies that the selective movement of fine grain-size fractions predominates in this system. In other words, where upstream transport rates are relatively high, fine materials are selectively flushed downstream more readily than at sites with lower upstream transport rates. Apparently, UPSTREAM reflects the degree of transport connectivity between upstream and downstream sites.

The results also indicate that channels draining granitic source areas tend to have finer mean grain sizes than those draining quartzite areas and that channels within arroyos generally have larger mean grain sizes than channels flanked by bedrock valley sides. The former result reflects the contribution of abundant amounts of fine-grained granodioritic grus to streams in the central portion of the basin. The latter result is due to the input of coarse material from unstable arroyo banks. Field observations indicated that weathered quartzite associated with bedrock outcrops consists of relatively small platy particles. Conversely, the arroyo banks contain large quartzite boulders that probably formed under a previous weathering regime. The contribution of these boulders to the channels at cutbanks produces local increases in mean grain size.

As expected, the degree of sorting (SORT) depends primarily on shear stress (STRESS) and the caliber of the bed material (MEAN). The standardized coefficients for Equation 12 indicate that MEAN is the dominant control. BANKTYPE and ROCKTYPE exert only weak influences on the degree of sorting. The positive sign for the ROCKTYPE coefficient is somewhat enigmatic; it suggests that the relatively coarse quartzite tends to be better sorted than the fine-grained granitic grus. The upstream sediment transport rate (UPSTREAM) apparently influences the degree of sorting only indirectly via MEAN.

At least two factors may be responsible for the positive relationship between MEAN and SORT. First, the grain size distributions of coarse parent materials will not be modified as easily by hydraulic forces as will those for fine parent materials. Second, microscale influences by
coarse particles (cobbles, boulders) tend to restrict entrainment and promote the deposition of small grains (coarse sand, fine gravel), thereby increasing the variance of the particle size distribution (Laronne and Carson 1976; Brayshaw 1985).

**Table 5. Extended**

<table>
<thead>
<tr>
<th>UPSTREAM BANKTYPE</th>
<th>ROCKTYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>.06</td>
<td>.42</td>
</tr>
</tbody>
</table>

**Implications for Dynamic System Response**

The structural process-response model represents an interpretation of the causal processes that generated the observed data. Although the structure of the model is based on a temporal cause-and-effect framework, it has been estimated with spatial rather than temporal data. In other words, location has been substituted for time. Location for time substitution is often inappropriate in geomorphology (Church and Mark 1980), but in some instances such a substitution may be valid (Paine 1985). In particular, the variables under consideration should change in a temporally self-similar fashion. Verification of this requirement is difficult, but the likelihood of its attainment is enhanced if the model is based on a sound conceptual framework, such as the Schumm and Lichty (1965) scheme used in this research (Paine 1985). If one assumes that the structural coefficients describe temporal as well as spatial relationships among the variables, then the estimated model embodies the dynamic behavior of the McDowell Mountain drainage system. One can speculate on the nature of this behavior by examining the total response of the system to a random disturbance. As an example, suppose a local site-specific process, such as a landslide, instantaneously delivers coarse sediment to a channel reach, increasing the mean grain size of the bed material. How will the system react to this exogenous disturbance?

Analysis of the system behavior entails two steps. The first step involves the reduced-form equations, which express each endogenous variable as a function of the exogenous variables only (Table 6). These equations indicate the total changes in the endogenous variables for changes in the exogenous variables (Hanushek and Jackson 1977, 257–58). They can be used to calculate the equilibrium value of each endogenous variable for constant values of the exogenous variables (Johnston 1984, 9). These equilibrium values assume that systematic exogenous forces are not influencing the system, i.e., that the error component of each reduced-form equation is zero. To evaluate the effect of a perturbation, the second step of the analysis involves tracing the direct and indirect effects of the change in the disturbed variable (in this case mean grain size) through the causal linkages of the channel sub-system. Because these linkages form a closed feedback loop, if any variable within this block changes, it must eventually indirectly affect itself.

Greene (1977) developed an algorithm for evaluating the total effects of endogenous variables on one another in nonhierarchical models. A total effects matrix \( E \) is computed as:

\[
E = I + M(0) + \sum_{i=2}^{k+1} M(0)^i
\]

where \( M(0) \) is a matrix of direct effect coefficients with zeros on the diagonal, \( I \) is a conformable identity matrix, \( i \) is the level of causal interaction among the endogenous variables, and \( M(0)^{k+1} \) is a null matrix. For log-transformed data, the coefficients in the total effects matrix indicate the total percentage change in each endogenous variable for unit percentage changes in the other endogenous variables. The direct effects at the initial level of causal interaction among the endogenous variables are provided by \( M(0) \), while the \( M(0)^i \) indicate the partial indirect effects at intermediate levels of interaction. Greene’s method will not always produce a null matrix for non-hierarchical systems over a finite number of interaction levels (\( i \)'s). The procedure does lead
Table 6. Estimated Reduced Form of Equations 7-11 (Table 3)

\[
\begin{align*}
\text{DISCH} &= 2.51 + .54(\text{AREA}) + .12(\text{RELIEF}) - .04(\text{UPSTREAM}) + .04(\text{BANKTYPE}) - .06(\text{ROCKTYPE}) \\
\text{SLOPE} &= -1.83 - .22(\text{AREA}) + .92(\text{RELIEF}) + .03(\text{UPSTREAM}) - .13(\text{BANKTYPE}) + .04(\text{ROCKTYPE}) \\
\text{WIDTH} &= .74 + .35(\text{AREA}) - .31(\text{RELIEF}) + .11(\text{UPSTREAM}) - .11(\text{BANKTYPE}) + .15(\text{ROCKTYPE}) \\
\text{STRESS} &= 7.15 - .09(\text{AREA}) + 1.04(\text{RELIEF}) - .01(\text{UPSTREAM}) - .09(\text{BANKTYPE}) - .01(\text{ROCKTYPE}) \\
\text{MEAN} &= 5.08 - .05(\text{AREA}) + .59(\text{RELIEF}) - .35(\text{UPSTREAM}) - .30(\text{BANKTYPE}) - .49(\text{ROCKTYPE})
\end{align*}
\]

to a solution for systems in which causal effects tend to become increasingly attenuated as they become increasingly indirect. Greene (1977) showed that for such systems \(E\) converges to \(\left[1 - M(0)\right]^{-1}\), a result that is well-known for dynamic models. Thus, the algorithm provides a link between dynamic and static system models.

The procedures described above were used to examine the dynamic response of the McDowell Mountain fluvial system to the postulated increase in mean grain size. The geometric means of \text{AREA}, \text{RELIEF}, and \text{UPSTREAM} were arbitrarily selected as constant values for these exogenous variables; it was also assumed that \text{BANKTYPE} is alluvial, \text{ROCKTYPE} is quartzite, and that sediment delivery increases mean grain size by 50 percent (Table 7). Equilibrium values for the endogenous variables were determined using the reduced-form equations (Table 6 and Table 7). Application of Greene’s algorithm to the matrix of direct effect coefficients for the channel subsystem (Table 8) indicated that by the end of the 16th interaction level the procedure had essentially converged on a null matrix. All indirect effects at this level were less than .0001. Calculation of \(E\) as \(\left[1 - M(0)\right]^{-1}\) confirmed the accuracy of the iterative solution (Table 8). The occurrence of the exogenous disturbance at the beginning of an interaction level seemed conceptually reasonable for a discontinuously-operating desert mountain fluvial system. Changes in the en-

<table>
<thead>
<tr>
<th>Table 7. Response of Variables in the Channel Subsystem to Exogenous Disturbance of Mean Grain Size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Values of Exogenous Variables:</strong></td>
</tr>
<tr>
<td><strong>WIDTH (m)</strong></td>
</tr>
<tr>
<td>Level 1</td>
</tr>
<tr>
<td>Level 2</td>
</tr>
<tr>
<td>Level 3</td>
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<tr>
<td>Level 4</td>
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<td>Level 11</td>
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<td>Level 13</td>
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<tr>
<td>Level 14</td>
</tr>
<tr>
<td>Level 15</td>
</tr>
<tr>
<td>Level 16</td>
</tr>
</tbody>
</table>
### Table 8. Direct Effects Matrix and Total Effects Matrix for Channel Subsystem

<table>
<thead>
<tr>
<th></th>
<th>DISCH</th>
<th>SLOPE</th>
<th>WIDTH</th>
<th>STRESS</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISCH</td>
<td>.00</td>
<td>-.74</td>
<td>.55</td>
<td>.40</td>
<td>.00</td>
</tr>
<tr>
<td>SLOPE</td>
<td>.00</td>
<td>.00</td>
<td>-.17</td>
<td>1.00</td>
<td>.00</td>
</tr>
<tr>
<td>WIDTH</td>
<td>-.39</td>
<td>.52</td>
<td>.00</td>
<td>-.22</td>
<td>.00</td>
</tr>
<tr>
<td>STRESS</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.00</td>
<td>.57</td>
</tr>
<tr>
<td>MEAN</td>
<td>.00</td>
<td>.16</td>
<td>-.38</td>
<td>.00</td>
<td>.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>DISCH</th>
<th>SLOPE</th>
<th>WIDTH</th>
<th>STRESS</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>DISCH</td>
<td>.80</td>
<td>-.33</td>
<td>.52</td>
<td>-.13</td>
<td>-.07</td>
</tr>
<tr>
<td>SLOPE</td>
<td>.11</td>
<td>.85</td>
<td>-.29</td>
<td>.96</td>
<td>.55</td>
</tr>
<tr>
<td>WIDTH</td>
<td>-.27</td>
<td>.58</td>
<td>.68</td>
<td>.33</td>
<td>.19</td>
</tr>
<tr>
<td>STRESS</td>
<td>.07</td>
<td>-.05</td>
<td>-.18</td>
<td>1.02</td>
<td>.58</td>
</tr>
<tr>
<td>MEAN</td>
<td>.12</td>
<td>-.09</td>
<td>-.31</td>
<td>.03</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Endogenous variables at each level of causal interaction subsequent to the disturbance were computed using the coefficients in row 5 of the direct and indirect effect matrices.

### Discussion

Results of the analysis show that all variables mutually adjust to the change in mean grain size via a damped series of positive and negative oscillations (Table 7, Figure 7). As expected, the system has essentially stabilized by the 16th interaction level. But the values of the endogenous variables after stabilization are different from their original, most probable states as determined from the reduced-form equations. The system apparently is metastable in that unspecified causes with positive or negative effects shift the equilibrium state for the disturbed variable upward (increase) or downward (decrease) respectively. The disturbed variable then changes the equilibrium states for the other endogenous variables via its direct impact on them. Subsequently, the variables converge on their new stable states through indirect feedback effects. Internal adjustments within the channel subsystem apparently do not lead to convergence of the variables on their original values, regardless of the magnitude of the random exogenous disturbance. Instead, these variables converge on a new set of stable values. This result implies that the system need not cross a geomorphic threshold in order to converge on a new equilibrium. Rather, switches in equilibria can arise due to continuous feedback interactions following random exogenous disturbances.

The analysis also suggests that there are a limitless number of stable states for any given set of values for the exogenous variables (LAG, RELIEF, AREA, BANKTYPE, ROCKTYPE). In other words, residuals in the reduced-form equations do not necessarily represent random departures from equilibrium or measurement errors. Rather these points may be alternative stable states that result from internal adjustments by the channel subsystem to seemingly random influences (i.e., unspecified causes). Some of the residuals may also represent sites that are in the process of adjustment, but that have not yet reached a stable condition. The attainment of a specific stable state depends on the number, magnitude, and temporal sequence of unsystematic disturbances of the system. In complex fluvial systems that are strongly affected by a wide variety of site-specific processes, such as the one examined here, the degree of scatter about the most probable stable state will be relatively large. It appears that extreme disturbances are required to attain stability conditions far removed from the most probable states. Despite the relatively large increase in mean grain size in the analysis described above, the values of the other endogenous variables after stability is achieved are not greatly different from their most probable stable states (Table 7). This response indicates that the mutual adjustment mechanisms are relatively inelastic. All exogenous processes will
produce some changes in the system, but these changes may be conspicuous only during extreme events.

It is unfortunate that the analysis does not provide insight into the time frame of the system response. The temporal length of the adjustment process probably depends on prevailing environmental conditions. It seems likely that adjustment occurs more rapidly in humid-temperate systems than in arid ones because recovery times of the latter are delayed by infrequent and discontinuous geomorphic processes (Wolman and Gerson 1978). If recovery times are so great that uninterrupted stabilization rarely occurs, the attainment of any stable state will be thwarted and short-term disequilibrium may predominate within the system (Figure 8). Documented evidence of alternating cut and fill sequences within arroyos of the Southwest (Patton and Schumm 1981; Graf 1983b) suggests that oscillatory adjustments, such as those described above, may be occurring in these fluvial systems.

The analysis of the system response is based on several assumptions. First, it assumes that the effects of a disturbance are transmitted throughout the feedback loop at equal rates, i.e., that the reaction periods represented by the causal links are all equal. In some situations, this assumption may be unrealistic. For example, geologic controls may limit adjustments in channel gradient or width. Under these conditions, certain causal pathways may operate with greater efficiency than others, resulting in behavior of greater complexity than that described above. The response will also be more complex if several endogenous variables are influenced simultaneously by an exogenous disturbance. In addition, the analysis assumes that the structural relationships among the variables in the model do not change as a result of the disturbance (i.e., that the system does not cross a geomorphic threshold) and that they are constant over time. The analysis of the system response represents a hypothesis concerning the dynamics of desert mountain drainage systems; it provides a comparative framework for future studies of temporal change in such systems. The need exists for long-term (10³-10⁵ years) studies that continuously monitor the states of geomorphic variables in desert mountain environments.

Conclusion

The development and analysis of a simultaneous-equation, process-response model for a typical desert mountain drainage system provides the basis for provisional answers to the research questions posed at the beginning of this paper. During the modern time span, desert mountain streams can be viewed as a subsystem controlled by the phys-
tical properties of the drainage basin. The structure of the stream channel subsystem consists of a closed feedback loop composed of mutually dependent variables.

In general, results confirm that discharge, channel width, channel slope, shear stress, and bed material caliber interact in an expected manner, although some of the causal linkages are rather weak. In particular, the influence of mean grain size on channel gradient is somewhat suspect, suggesting that the association between gradient and grain size in this system primarily reflects the indirect effect of gradient on grain size (via shear stress), rather than the direct effect of grain size on gradient. The grain size-gradient relationship needs to be explored in a variety of fluvial systems to determine the exact nature of the causal association between these two variables.

The movement of relatively fine grain-size fractions appears to predominate in this desert mountain fluvial system. This conclusion is based on a negative association between the sediment transport rate at a given site and the mean grain size at the site immediately downstream. Brierly and Hickin (1985) and Miller (1958) reached similar conclusions regarding sediment movement in high-energy mountain streams of British Columbia and New Mexico respectively. Perhaps this phenomenon is a general characteristic of mountain streams.

Analysis of the system's response to a random disturbance indicates that this desert mountain drainage is self-regulatory, but its behavior differs somewhat from the classic concept of steady-state equilibrium (Schumm and Lichty 1965; Knighton 1984, 90–91). When an endogenous variable is displaced from its initial, most probable condition by an unsystematic exogenous disturbance, all of the variables in the channel subsystem respond by converging on new limits; they do not fluctuate about or return to their initial values. The structural relationships in the model suggest that a random disturbance of any magnitude will result in oscillatory convergence on a new set of stable values, i.e., the system is metastable and need not exceed a geomorphic threshold in order for the variables to attain new equilibrium states. Slingerland (1981) reached a similar conclusion via qualitative stability analysis of a dynamic stream system model. This type of behavior most closely exemplifies the concept of equilibrium proposed by Howard (1982).

This study demonstrates the utility of simultaneous-equation models for explicitly analyzing mutual adjustment mechanisms within geomorphic systems. These models also promise insight into theories concerning system structures and dynamics. But it should be noted that this approach is subject to the limitation of all multivariate statistical methods, viz., that the fitted relationships may not extend beyond the range of conditions studied. The model developed in this research is based on observations of a temporally-static fluvial system; it represents the fundamental level of simultaneous systems analysis. The challenge lies ahead to develop dynamic simultaneous-equation models of earth surface systems based on actual observations of temporal change.

Acknowledgments

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Notes

1. Consistency is a desirable property of an estimator wherein it converges on the true value as the sample size becomes infinitely large.
2. The t-tests for the nonhierarchical equations (7–11) are only approximate because the distributions of the coefficients calculated by two-stage least squares are only approximately normal (Christ 1966, 515–16). Likewise, the $R^2$ statistic is not strictly comparable to the $R^2$ for ordinary least squares. In particular, $R^2$ for simultaneous-equations models does not have a lower bound of zero (Judge et al. 1982, 147; Hanushek and Jackson 1977, 269).
3. Estimates of the reduced-form coefficients can be computed as $\hat{B} = \hat{B} \cdot \hat{\Gamma}$ where $\hat{B}$ is the matrix of structural coefficients for the endogenous variables and $\hat{\Gamma}$ is the matrix of structural coefficients for the exogenous variables (Johnston 1984, 477).
4. Goldberg (1958, 237–38) showed that a necessary and sufficient condition for the convergence of the coefficient matrix is that the absolute value of the largest eigenvalue of this matrix must be <1.
References


