MA 350 Dr. G. Stoudt
Fifth Reading Assignment

Readings

- Biography of Eudoxus, page 74
- Reading 22: From Book V of the *Elements*: Definitions and Propositions 1, 7, 9, and 10 (Theory of Proportions)-Eudoxus/Euclid
- Reading 23: From Book XII.2 of the *Element*s (Method of Approximation, the So-called Method of Exhaustion)-Euclid

Notes for the Readings

Reading 22

You must think ratios, not fractions: a : b = c : d.

Reading 23

On page 80, the statement that "the polygon AOBPCQDR similar to the polygon EKFLGMHN; therefore as the square on BD is to the square on FH, so is the polygon AOBPCQDR to the polygon EKFLGMHN." is from Elements XII.1: Similar polygons inscribed in circles are to one another as the squares on the diameters.

Questions for Discussion

Reading 22

1. Can there be a ratio between the area of a circle and its radius? What about the area of a circle and the area of a polygon?
2. "Circles are to one another as the squares on the diameters." What does that mean algebraically?
3. Using variable letters and ratio notation, what does Definition 5 say?
4. Using variable letters and ratio notation, what does Proposition 7 say?
5. Using variable letters and ratio notation, what does Proposition 9 say?
6. Using variable letters and ratio notation, what does Proposition 10 say?
7. In the proof of Proposition 7, considering that he needs to use Definition 5, explain why the proof goes the way it does.

Reading 23

We will dissect this reading rather carefully, so read it carefully and bone up on your geometry!

Homework Problems

Reading 23

1. Write up the given proof (using modern terminology and symbols if you like) of "the [area of the] inscribed square EFGH is greater than half [the area] of the circle EFGH."
2. Write up the given proof (using modern terminology and symbols if you like) of "the [area of the] triangle EKF is greater than half [of the area] of the segment of the circle [cut off by] EKF."
3. Consider the statement from Elements X.1 "if two unequal magnitudes be set out, and if from the greater there be subtracted a magnitude greater than the half, and from that which is left a greater than the half, and if this is done continually, there will be left some magnitude which will be less than the lesser magnitude set out." It is mentioned after X.1 in the Elements that this is true "even if the parts subtracted be halves." Using "halves" instead of "greater than the half" show why this is equivalent to the following:

   Let a be some real number and let $\varepsilon > 0$ be given. Then there exists some positive integer n such that $\frac{a}{2^n} < \varepsilon$.