Appendix

Table of Standards and Expectations

With labeling for lesson plans:

In your lesson plan, you may label the standard you are addressing as, for example,

Number and Operations, H. 2. c.

Please refer to the complete text of Principles and Standards for School Mathematics in the Secondary Lab, from your instructor, or online at http://standards.nctm.org/

Number and Operations

STANDARD	M	H
Instructional programs from prekindergarten through grade 12 should enable all students to—	**Grades 6–8** Expectations In grades 6–8 all students should—	**Grades 9–12** **Expectations** In grades 9–12 all students should—
1. Understand numbers, ways of representing numbers, relationships among numbers, and number systems	**a.** • work flexibly with fractions, decimals, and percents to solve problems; **b.** • compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line; **c.** • develop meaning for percents greater than 100 and less than 1; **d.** • understand and use ratios and proportions to represent quantitative relationships; **e.** • develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; **f.** • use factors, multiples, prime factorization, and relatively prime numbers to solve problems; **g.** • develop meaning for integers and represent and compare quantities with them.	**a.** • develop a deeper understanding of very large and very small numbers and of various representations of them; **b.** • compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions; **c.** • understand vectors and matrices as systems that have some of the properties of the real-number system; **d.** • use number-theory arguments to justify relationships involving whole numbers.
2. Understand meanings of operations and how they relate to one another	**a.** • understand the meaning and effects of arithmetic operations with fractions, decimals, and integers; **b.** • use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals; **c.** • understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.	**a.** • judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities; **b.** • develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices; **c.** • develop an understanding of permutations and combinations as counting techniques.
3. Compute fluently and make reasonable estimates	**a.** • select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods; **b.** • develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use; **c.** • develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results; **d.** • develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.	**a.** • develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases. **b.** • judge the reasonableness of numerical computations and their results.

Algebra
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

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<th>M
Grades 6–8
Expectations
In grades 6–8 all students should–</th>
<th>H
Grades 9–12
Expectations
In grades 9–12 all students should–</th>
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<tr>
<td>1. Understand patterns, relations, and functions</td>
<td>a. • represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
b. • relate and compare different forms of representation for a relationship;
c. • identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.</td>
<td>a. • generalize patterns using explicitly defined and recursively defined functions;
b. • understand relations and functions and select, convert flexibly among, and use various representations for them;
c. • analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d. • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
e. • understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
f. • interpret representations of functions of two variables.</td>
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<td>2. Represent and analyze mathematical situations and structures using algebraic symbols</td>
<td>a. • develop an initial conceptual understanding of different uses of variables;
b. • explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;
c. • use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
d. • recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.</td>
<td>a. • understand the meaning of equivalent forms of expressions, equations, inequalities, and relations;
b. • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency–mentally or with paper and pencil in simple cases and using technology in all cases;
c. • use symbolic algebra to represent and explain mathematical relationships;
d. • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations;
e. • judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.</td>
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<td>3. Use mathematical models to represent and understand quantitative relationships</td>
<td>a. • model and solve contextualized problems using various representations, such as graphs, tables, and equations.</td>
<td>a. • identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
b. • use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
c. • draw reasonable conclusions about a situation being modeled.</td>
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<td>4. Analyze change in various contexts</td>
<td>a. • use graphs to analyze the nature of changes in quantities in linear relationships.</td>
<td>a. • approximate and interpret rates of change from graphical and numerical data.</td>
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Geometry

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Grades 6–8

Expectations

In grades 6–8 all students should–

Grades 9–12

Expectations

In grades 9–12 all students should–

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1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

a. • precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;

b. • understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;

c. • create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

a. • analyze properties and determine attributes of two- and three-dimensional objects;

b. • explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

c. • establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;

d. • use trigonometric relationships to determine lengths and angle measures.

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2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

a. • use coordinate geometry to represent and examine the properties of geometric shapes;

b. • use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

a. • use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;

b. • investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

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3. Apply transformations and use symmetry to analyze mathematical situations

a. • describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;

b. • examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

a. • understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices;

b. • use various representations to help understand the effects of simple transformations and their compositions.

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4. Use visualization, spatial reasoning, and geometric modeling to solve problems

a. • draw geometric objects with specified properties, such as side lengths or angle measures;

b. • use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;

c. • use visual tools such as networks to represent and solve problems;

d. • use geometric models to represent and explain numerical and algebraic relationships;

e. • recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

a. • draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;

b. • visualize three-dimensional objects from different perspectives and analyze their cross sections;

c. • use vertex-edge graphs to model and solve problems;

d. • use geometric models to gain insights into, and answer questions in, other areas of mathematics;

e. • use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Measurement

STANDARD *Instructional programs from prekindergarten through grade 12 should enable all students to—*	M Grades 6–8 Expectations In grades 6–8 all students should–	H Grades 9–12 Expectations In grades 9–12 all students should–
1. Understand measurable attributes of objects and the units, systems, and processes of measurement	a. • understand both metric and customary systems of measurement; b. • understand relationships among units and convert from one unit to another within the same system; c. • understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.	a. • make decisions about units and scales that are appropriate for problem situations involving measurement.
2. Apply appropriate techniques, tools, and formulas to determine measurements	a. • use common benchmarks to select appropriate methods for estimating measurements; b. • select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision; c. • develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes; d. • develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders; e. • solve problems involving scale factors, using ratio and proportion; f. • solve simple problems involving rates and derived measurements for such attributes as velocity and density.	a. • analyze precision, accuracy, and approximate error in measurement situations; b. • understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders; c. • apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations; d. • use unit analysis to check measurement computations.

Data Analysis and Probability

STANDARD	M Grades 6–8	H Grades 9–12
Instructional programs from prekindergarten through grade 12 should enable all students to—	Expectations In grades 6–8 all students should–	**Expectations** In grades 9–12 all students should–
1. Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	a. • formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; b. • select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.	a. • understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each; b. • know the characteristics of well-designed studies, including the role of randomization in surveys and experiments; c. • understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable; d. • understand histograms, parallel box plots, and scatterplots and use them to display data; e. • compute basic statistics and understand the distinction between a statistic and a parameter.
2. Select and use appropriate statistical methods to analyze data	a. • find, use, and interpret measures of center and spread, including mean and interquartile range; b. • discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.	a. • for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics; b. • for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools; c. • display and discuss bivariate data where at least one variable is categorical; d. • recognize how linear transformations of univariate data affect shape, center, and spread; e. • identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.
3. Develop and evaluate inferences and predictions that are based on data	a. • use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken; b. • make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit; c. • use conjectures to formulate new questions and plan new studies to answer them.	a. • use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions; b. • understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference; c. • evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions; d. • understand how basic statistical techniques are used to monitor process characteristics in the workplace.
4. Understand and apply basic concepts of probability	a. • understand and use appropriate terminology to describe complementary and mutually exclusive events; b. • use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; c. • compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.	a. • understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases; b. • use simulations to construct empirical probability distributions; c. • compute and interpret the expected value of random variables in simple cases; d. • understand the concepts of conditional probability and independent events; e. • understand how to compute the probability of a compound event.

Problem Solving
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

a. • Build new mathematical knowledge through problem solving
b. • Solve problems that arise in mathematics and in other contexts
c. • Apply and adapt a variety of appropriate strategies to solve problems
d. • Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

a. • Recognize reasoning and proof as fundamental aspects of mathematics
b. • Make and investigate mathematical conjectures
c. • Develop and evaluate mathematical arguments and proofs
d. • Select and use various types of reasoning and methods of proof

Communication
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

a. • Organize and consolidate their mathematical thinking through communication
b. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
c. • Analyze and evaluate the mathematical thinking and strategies of others
d. • Use the language of mathematics to express mathematical ideas precisely

Connections
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

a. • Recognize and use connections among mathematical ideas
b. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
c. • Recognize and apply mathematics in contexts outside of mathematics

Representation
STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

a. • Create and use representations to organize, record, and communicate mathematical ideas
b. • Select, apply, and translate among mathematical representations to solve problems
c. • Use representations to model and interpret physical, social, and mathematical phenomena